Fear and Laughing of the Market: trending pessimism, fragile optimism

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Abstract

Building on fractional Black-Scholes, this paper draws a connection between option implied Hurst exponent H and current market mood. H comes with several advantages over VIX and survey based sentiment measures, such as a straight forward interpretation (*optimistic/pessimistic*) or the highly frequent data availability. From global evidence, we find strong correlations of H with a broad range of well established sentiment gauges. Analyzing H in more detail, investor fear occurs much faster than confidence is gained back. From periodical plus rolling long-term memory analysis for eight major regions around the globe, we observe that persistence in sentiment is depended on its level: the higher the market mood, the more overreacting and fragile it becomes. Other way around, if pessimism rises, then the mood gets stable and trending.

Keywords: Market Mood, Investor Sentiment, Implied Volatilities, Long-Term Memory, Fractal Analysis

JEL: G01, G12, G4, G15

1. Introduction

There is an old saying on Wall Street: Only two things drive the markets - fear and greed. Perhaps this why market inefficiencies are frequently claimed to be a natural consequence of investor emotions. Shefrin [2000] for example claim that such behavioral driven anomalies can be observed at different markets and asset classes, or Shiller [2003] who points out the importance of psychological effects describing price movements beyond fundamental explanations. From the large body of behavioral literature, investor sentiment is given an important role.[1] Respectively, a famous and widely used market fear gauge is the CBOE Volatility Index (VIX) (e.g. in Whaley [2000] or Baker and Wurgler [2007]), which, however, does not allow to make a clear interpretation whether investors are bullish, neutral or bearish. In difference, surveys directly measure the proportion of optimistic investors to pessimistic ones, but come with drawbacks of less frequency, extensive effort

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[1] We use the wording *sentiment* in a directional sense, meaning that it describes investor mood of being bullish or bearish. This should not be mistaken with studies taking sentiment as un-directional investor attention, the CBOE VIX would be an example of such unidirectional sentiment.

in collecting data and the impossibility of capturing the complete market picture. To overcome the problems from both approaches, we promote the option implied Hurst exponent H to be a valid candidate for quantifying actually traded investor sentiment - straight forward to interpret and easy to estimate at almost continuous frequency. From the fractional Black-Scholes model (cp. Hu and Øksendal [2003]), we formulate a hypothesis that allows us to interpret H as sentiment.

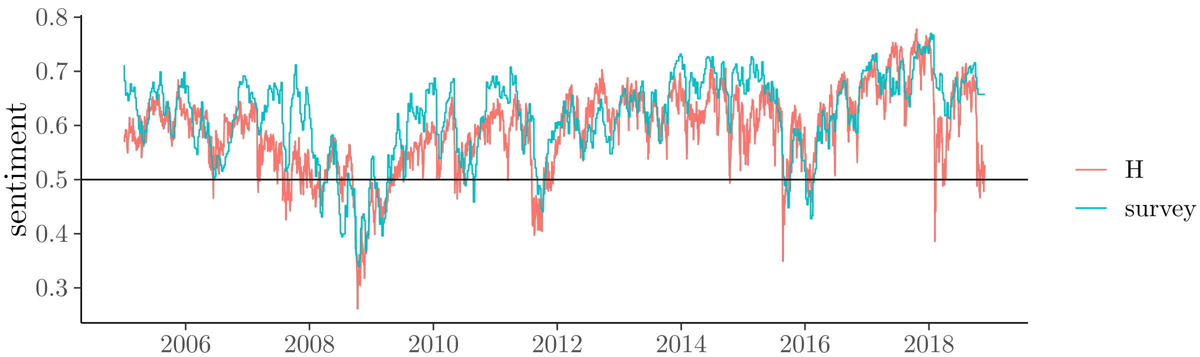

Figure 1
Visualization of U.S. implied Hurst exponent H (daily) and a survey based sentiment measure (weekly)[2]. Both measures vary between 0 and 1 with 0.5 as the cut-off level. Above 0.5 means investor sentiment is bullish, below 0.5 indicates bearish. As it can be seen, H and the survey based measure share very similar movements, which supports our hypothesis that H reflects sentiment. Similar to the VIX, H comes at high data frequency and is derived from implied volatilities. Different to the VIX it allows to distinguish between bullish/bearish mood and comes with theoretical reasoning of why it actually represents sentiment.

Empirically, analyzing eight different regions around the globe, we find that H significantly correlates with most other sentiment gauges such as volatility indices (VX),[3] investor surveys, consumer confidence or other relevant measures. This pattern gives robustness for H to reflect the current market mood. Investigating daily H - observed from implied volatility term structures of equity indices - we find that investor sentiment is significantly negatively skewed, such that fear occurs way faster than confidence is gained back. Applying long-term memory analysis confirms this interpretation, as we find strong evidence that persistence of investor sentiment depends on the current level of market mood, such that during bearish times sentiment shows trending behavior, while in outside crises periods investor optimism is characterized by anti-persistence and overreactions. So to say, market fear is a mood that is trending and persistent, but confidence is fragile and instable. Robustness of our results is given by global evidence, use of several test settings, application of different long-term memory measures and from substituting H with country volatility indices. From these findings, we believe this paper to be of interest for a broad range finance scholars and investment practitioners.

[2]Source: Adivsor's Sentiment Report, Investors Intelligence; $sentiment = 1 \cdot \%bullish + 0.5 \cdot \%neutral + 0 \cdot \%bearish$
[3]VIX is a registered trademark from the Cboe Global Markets Inc. Similar volatility indices for other countries are abbreviated as VX

The reading is set up as follows. Section 2 introduces the theoretical framework which allows us to interpreted the implied Hurst exponent as market sentiment. Section 3 continues with a description of the input data and relevant settings to then present our empirical findings. Section 4 concludes. Robustness tests and complementary material are attached at the Appendix.

2. Concept of *H* being Sentiment

The idea we present within this paper builds on equity's option implied volatility term structure and sketches a link to investor sentiment.[4] Given markets are information efficient (Fama [1970]), investor expectations are reflected not only within the current stock price, but also at corresponding options. While for a given point in time stocks only have one data point of price, there exist multiple options for the very same underlying, differing in strikes and maturities. These option prices - covering forward looking information - can be rewritten to express the risk, i.e. volatilities, they are priced under. A large body of literature exists discussing such option implied volatility surfaces, its risk-neutral moments and how they affect underlying equity returns. Most of such studies take the perspective on options with fixed target maturity and varying degrees of moneyness.[5] Fewer academic papers apply a decomposition of the implied volatility surface over maturities[6] - which we refer to as the volatility term structure. Perhaps the most popular example of implied volatilities is the CBOE Volatility Index - or short - VIX, constructed from options with fixed target maturity of one month but distinguishing in strikes. The VIX is broadly accepted to reflect current market fear, say if the VIX is high, then investors are said to be nervous and vice versa for the other way around (cp. Whaley [2000]; Baker and Wurgler [2007] or Caporale et al. [2018]). However, the disadvantage under the VIX arises, that there is no clear interpretation in a directional sense - there are periods where the index is generally higher and times where it is basically lower. This makes it hard to distinguish whether the market mood is currently bullish or bearish - an important distinction which many practitioners and academics may be interested in.

In a nutshell, our hypothesis claims that if the implied volatility term structure is upward sloping, then investor sentiment is optimistic, if it is flat then the mood is neutral, and decreasing over maturity is understood as pessimism. Figure 2 displays the volatility term structure for the SP500 at two different points in time. Technically, we apply fractal theory to derive the implied

[4]Basically, this work extends the initial idea of Schadner [2019] and provides empirical insights on sentiment behavior.

[5]To name a few examples, Jarrow and Rudd [1982]; Backus et al. [2004] or Mixon [2010] suggest estimation methods to measure implied moments. Bali et al. [2017] show how expected volatility, skewness and kurtosis relates to expected returns.

[6]Examples are Li and Chen [2014]; Flint and Mare [2016].

Hurst exponent H - ranging between zero and one - which exactly does this indication of whether the curve is increasing over maturity ($H > 0.5$; *bullish*), flat ($H = 0.5$; *neutral*) or downward sloping ($H < 0.5$; *bearish*).

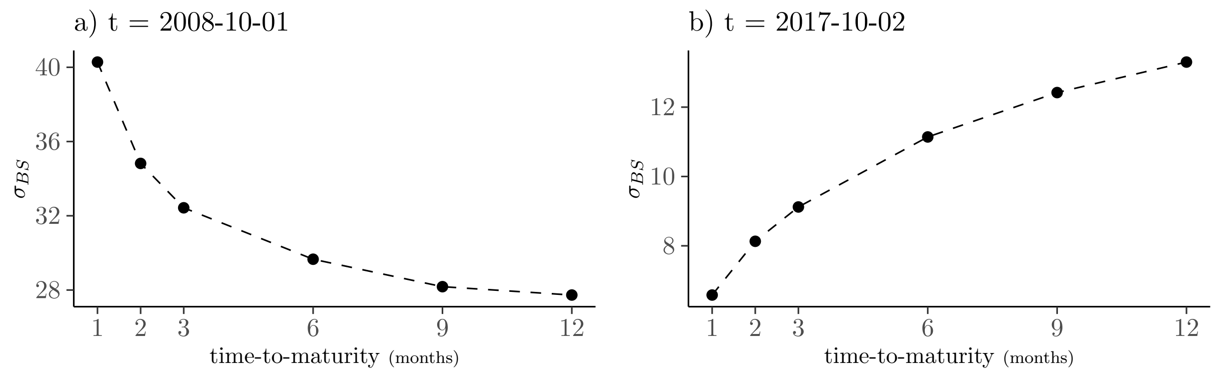

Figure 2
Option implied Black-Scholes volatility term structure (σ_{BS}; at-the-money) of the SP500 for two different days, once within the period of the financial crises (left) and once during a growing market phase (right). Should be flat under classical Brownian motion and no-momentum assumptions, but rarely the case within empirical data. VIX measures only the level of the one-month ahead data point; differently, H indicates the slope of the entire term structure.

2.1. Literature on fractal Brownian Motion in Finance

Kolmogorov [1940] was actually the founder of fractal Brownian motion, but it became popular through Hurst [1956] and Mandelbrot and Van Ness [1968]. The key idea behind this concept is that it allows a time series to show autocorrelation, depended on the Hurst parameter H. If H indicates positive auto-correlation, then the series will be persistent and faces long term memory. Vice versa, the case where H indicates negative auto-correlation, the process will be anti-persistent with properties of short term memory. These features of long and short term memory made fractal Brownian motion interesting for both empirical and theoretical research in finance. Empirical studies typically use the fractal concept to analyze persistence within return series, for example Peters [1991, 1994] finds evidence of such long-term memory within U.S. stock returns. Or Granger and Ding [1995], who detect return persistence at the S&P500 index. Alvarez-Ramirez et al. [2008] and Dominique and Rivera-Solis [2011] further extend this work to show that persistence in S&P500 returns varies over time, and is especially different during crisis times. Beyond the application on return series, another example would be Caporale et al. [2018], who examine that also the persistence of market fear - measured by the VIX - was greater during the financial crises. Fractal Brownian motion also found its way into the theoretical asset pricing literature. Hu and Øksendal [2003] and Elliott and Van Der Hoek [2003] developed the fractional Black-Scholes model, where the self-financing portfolio is defined as the risk less bank account and some risky asset driven by fractal Brownian motion. Their model initiated an discussion among finance academics whether this market model is truly arbitrage free or not, e.g. Björk and Hult [2005] were ones to bring in this critique. Cheridito [2003], however, points out that these arbitrage opportunities would only exist if investors could trade infinitely fast, which is obviously not the case in real world markets. Further modeling building on fractional Black-Scholes can be found in Mishura [2004], Liu and Chang [2013], Jiménez and Martínez [2017] or Garnier and Solna [2017], who basically extend this framework of Hu and Øksendal [2003]. Also interesting, Rostek and Schöbel [2010] put the Hu and Øksendal [2003] model into a perspective of a consumption based decision problem, which allows a similar estimation procedure for H as in the fractional Black-Scholes model, but overcomes the theoretical arbitrage opportunity. Besides asset pricing, according to Morelli and Santucci de Magistris [2019], fractional processes are also a popular framework to model volatility dynamics, which points out the potential of fractal Brownian motions for risk management purposes. Further studies using fractal Brownian motion to model volatility processes directly are **?** or **?**.

2.2. The Model of Expected Momentum and Market Sentiment

Suggest a fractional Black-Scholes market as defined in Hu and Øksendal [2003]. Let B^H denote a fractal Brownian motion, taken under the real probability measure $\mathbf{P}$. Generally, a fractal Brownian motion is characterized by the mean

$$\mathbb{E}[B_t^H] = 0 \qquad \forall t \in \mathbb{R} \quad (1)$$

but different to the classical form, the covariance among increments is given through

$$\mathbb{E}[B_t^H\, B_s^H] = \frac{1}{2}\left[|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right] = cov(B_t^H, B_s^H) \qquad \forall t, s \in \mathbb{R} \quad (2)$$

so H is the determinant of the process' autocorrelation. In the case of $H = 0.5$, this reduces to

$$cov(B_t^H, B_s^H) = Var(B_t^H) = t = Var(B_t) \qquad \text{iff } H = 0.5,\ t \leq s \quad (3)$$

which would generate the common form of classic Brownian motion B_t, where variance scales in time by factor t. However, if $H > \frac{1}{2}$ then $cov(B_t^H, B_s^H) > t$ and the time series is said to realize positive autocorrelation and persistence. Vice versa, for $H < \frac{1}{2}$ one observes a negatively autocorrelated and anti-persistent process having $cov(B_t^H, B_s^H) < t$. Therefore, in simple words, for $H \neq \frac{1}{2}$ fractal Brownian motion allows for (anti-)persistence through variance scaling larger or less than t.

This machinery can now brought into asset pricing through the fractional Black-Scholes model. Consider a market of two assets: a risk less bank account A

$$dA_t = r\, A_t\, dt, \qquad A_0 = 1, \qquad t \in [0, T] \quad (4)$$

and a risky asset S, driven by the fractal process

$$dS_t = \mu S_t \, dt + \sigma S_t \, dB_t^H, \qquad S_0 > 0, \tag{5}$$

with constant drift μ, volatility σ and the risk less rate of return r. Note, under this framework, S_t can be expressed as

$$S_t = S_0 \, \exp\left(\mu \, t - \frac{1}{2}\sigma^2 t^{2H} + \sigma B_t^H\right) \tag{6}$$

(cp. Elliott and Van Der Hoek [2003]). In our setting, we simply think of S as the equity market portfolio. To derive the fractional Black-Scholes model, Hu and Øksendal [2003] formulate the price process of the self-financing portfolio Z,

$$dZ_t = u_t \, dA_t + v_t \, dS_t \tag{7}$$

which replicates a contingent claim at maturity T, with u_t and v_t as the weights invested in A and S respectively. Equivalently to a standard Black-Scholes market, to make dZ_t a valuation process it has to be taken under risk-neutrality. Hereby, Hu and Øksendal [2003] show that Girsanov theorem is applicable such that B^H can be taken under a new measure $\mathbf{Q}$, which gives the risk-neutral process $\hat{B}^H$:

$$\hat{B}_t^H = B_t^H - \int_0^t \phi_s \, ds \tag{8}$$

with ϕ as the *market risk premium* or *market price of risk*. Under the $\mathbf{Q}$ measure, dS_t will take the form of

$$dS_t = rS_t \, dt + \sigma S_t \, d\hat{B}_t^H \tag{9}$$

which can now be used for dZ_t to give the valuation process used within fractional Black-Scholes. More precisely, according to Elliott and Van Der Hoek [2003] this transformation is enabled if we define ϕ_t as

$$\phi_t = \left(\frac{\mu - r}{\sigma}\right) \cdot \left[(T-t)^{H-\frac{1}{2}} + t^{H-\frac{1}{2}}\right] \tag{10}$$

which we believe to have some interesting implications. Let $\tau = T - t$ define the investor's forward looking horizon and suggest we are at some given point in time t, at which we observe the current expectation on H. With fixed t, we drop this subscript for the notation below and focus on ϕ over different maturities. Note that within classic finance models like Black-Scholes or standard consumption based asset pricing, ϕ is τ-independent and thus flat among the horizon, however, with allowance for autocorrelation in returns, fractal Brownian motion releases this τ-independence. Therefore, from the fractal model point of view, ϕ becomes a function of τ with some interesting interpretability. To understand how $\phi(\tau)$ behaves under different levels of H, we take the derivative of Eq. 10 with respect to τ:

$$\frac{\partial \phi(\tau)}{\partial \tau} = (H - \tfrac{1}{2}) \cdot \left(\frac{\mu - r}{\sigma} \right) \cdot \tau^{H - \frac{3}{2}} \tag{11}$$

With a special focus on $(H - \frac{1}{2})$, we now see that H is a crucial determinant of how the market price of risk is expected to evolve for the future τ ahead. Recap that H indicates auto-correlation and the market risk premium is ex-ante positively defined. Given a common way in finance is to define *ex-post* equity momentum as autocorrelation in returns, we think it makes sense to speak of (*ex-ante* or) *risk-neutral* momentum in the fractal framework: if implied H (taken under the $\mathbf{Q}$ measure) indicates positive ex-ante autocorrelation on the positively defined market risk premium, then $\phi(\tau)$ is believed to grow for the future ahead. This is what we understand as positive risk-neutral momentum. More formally, if $H > 0.5 \implies \frac{\partial \phi(\tau)}{\partial \tau} > 0 \implies$ positive risk-neutral momentum. The other way around, $H < 0.5$ would thus mean $\frac{\partial \phi(\tau)}{\partial \tau} < 0$ so $\phi(\tau)$ is expected to decline in τ, i.e. negative risk-neutral momentum. In the case of standard Brownian motion having $H = 0.5$, investors put a constant market risk premium for all horizons which results in $\frac{\partial \phi(\tau)}{\partial \tau} = 0$, so the market currently trades at an equilibrium of no momentum believes.

With this in mind, we recognize Eq. 11 in the sense that if investors expect positive momentum, then they are in a optimistic mood. If believes are in an equilibrium of negative momentum, they are pessimistic, and no risk-neutral momentum would equal neutral market sentiment. Fig. 3 visualizes this relation.

$$H \begin{cases} > 0.5 \implies \underset{\textit{ex-ante}}{+ \text{ auto-cor.}} \implies \frac{\partial \phi(\tau)}{\partial \tau} > 0 \implies \underset{\phi(\tau)\ \textit{expect. to increase}}{+ \text{ R.N. Momentum}} \implies \text{bullish} \\ = 0.5 \implies \underset{\textit{ex-ante}}{\text{no auto-cor.}} \implies \frac{\partial \phi(\tau)}{\partial \tau} = 0 \implies \underset{\phi(\tau)\ \textit{expect. to remain}}{\text{no R.N. Momentum}} \implies \text{neutral} \\ < 0.5 \implies \underset{\textit{ex-ante}}{- \text{ auto-cor.}} \implies \frac{\partial \phi(\tau)}{\partial \tau} < 0 \implies \underset{\phi(\tau)\ \textit{expect. to decrease}}{- \text{ R.N. Momentum}} \implies \text{bearish} \end{cases}$$

Figure 3
Option implied Hurst exponent H as a measure of market sentiment: ex-ante autocorrelation upon the market risk premium $\phi(\tau)$ determines risk-neutral momentum, which we interpret as investor sentiment. Similar to the CBOE VIX, this measure can be computed on an almost continuous frequency and captures the picture of the entire market. Different to the VIX, it can be interpreted in a directional sense (bullish/bearish) just like a survey based sentiment measure and further comes with theoretical reasoning of why it actually represents market sentiment.

Interestingly, $H \in (0, 1)$ implies $(H - \frac{3}{2})$ to be strictly negative. Thus, from taking limits of Eq.

11 we see

$$\lim_{\tau\to\infty} \frac{\partial \phi_t(\tau)}{\partial \tau} \to 0, \quad \forall H \qquad\qquad \text{given that} \quad \frac{\partial^2 \phi(\tau)}{\partial \tau^2} \begin{cases} > 0, \text{ for } H < 0.5 \\ < 0, \text{ for } H > 0.5 \end{cases} \tag{12}$$

Since per definition $\mu \geq r$, the range of $\phi(\tau)$ is determined as

$$\phi(\tau) \in (0, +\infty) \tag{13}$$

meaning that at time t, $\phi_t(\tau)$ is bounded between zero and infinity, but can never reach these limits. From an economic point of view, this makes sense as $\phi(\tau)$ can neither become negative nor explode.

Remarks on the Expectation Hypothesis

One may criticize that on average the expectation hypothesis, i.e. that implied volatilities equal expected volatilities, fails due to the existence of a variance risk premium VRP.[7] Recent literature throughly finds empirical support for an existence of such a VRP, e.g. Carr and Wu [2006, 2009]; Drechsler [2013] or Fassas and Papadamou [2018]. Given nowadays there are many possibilities to directly trade implied volatilities (e.g. VIX futures), it seems plausible that investors put a premium on those contracts as they face volatility-of-volatility risk. If a VRP exists, then, according to Carr and Wu [2009] or Fassas and Papadamou [2018], implied σ from option data has to be separated into the expected volatility $\mathbb{E}[\sigma]$ and the VRP:[8]

$$\sigma = \mathbb{E}[\sigma] + VRP \tag{14}$$

Hence, if we substitute this back into Eq. 10, we see that by separating the volatility into its expectational component and its VRP-component, existence of a VRP will not change the mechanism we derived from Eq. 11 & Fig. 3, such that

$$\text{sgn}\left(\frac{\partial \phi(\tau)}{\partial \tau}\right) \perp\!\!\!\perp VRP. \tag{15}$$

Note that (standard) Black-Scholes implied volatilities σ_{BS} can be easily connected to the fractal framework through

$$\sigma_{BS} = \sigma \cdot \tau^{H-\frac{1}{2}} \tag{16}$$

[7] This was early discussed in Campa and Chang [1995] which initialized flourishing research in that area.

[8] One may think of VRP as $VRP = \gamma \cdot sd(\mathbb{E}[\sigma])$ with $\gamma \geq 0$ as risk aversion and $sd(\mathbb{E}[\sigma])$ as volatility-of-volatility. This relation should help to get the idea we are drawing and is not further discussed as its relevance is neglectable in our framework.

(cp. Hu and Øksendal [2003]; Li and Chen [2014]). Consequently, σ_{BS} can be decomposed into the un-autocorrelated base level of volatility σ which scales by $\tau^{H-\frac{1}{2}}$ to correct for auto-correlation in the underlying returns. When accounting for VRP, this becomes

$$\sigma_{BS} = (\mathbb{E}[\sigma] + VRP) \cdot \tau^{H-\frac{1}{2}}, \tag{17}$$

from which implied H can be estimated. Thus, given the estimation method we introduce below (Eq. 18), existence of a VRP may bias the estimate for σ, but will have no influence on our sentiment measure H. Therefore, we suggest that the variance risk premium is of no importance when estimating H from implied Black-Scholes volatilities. So to speak, the expectation hypothesis may not hold, but we still argue that implied H is identical with expected H. As a consequence, the conclusion of our model does not change after correcting for VRP.

3. Empirical Evidence

Technically, we estimate H from simple OLS regression of the at-the-money log Black-Scholes volatility term structure following Hu and Øksendal [2003]:[9]

$$\underbrace{\ln(\sigma_{BS})}_{\hat{y}} = \underbrace{\ln(\sigma)}_{\hat{\alpha}} + \underbrace{(H - 0.5)}_{\hat{\beta}} \cdot \underbrace{\ln(\tau)}_{x} + \epsilon \tag{18}$$

such that

$$\sigma = e^{\hat{\alpha}} \qquad \text{and} \qquad H = \hat{\beta} + 0.5 \tag{19}$$

To ensure precise replication, we set option's strike to at-the-money as in such a case the underlying's expected payoff is replicated by 1:1 relation of Puts and Calls,[10] which should capture risk neutral skewness and kurtosis effects on the underlying equity index.

3.1. Data and Research Setup

All data used is derived from Thomson Reuter's Datastream and Bloomberg L.P. Due to the availability and quality of option data, our observation horizon starts in 2007,[11] focusing on main equity indices from seven countries with large market capitalization plus the Euro-Zone as a whole represented by the EuroSoxx50.[12] In order to enable robustness tests, we require markets to also have volatility indices available. For every equity index, H is estimated according to Equation

[9]Other empirical studies applying this estimation method are for example Li and Chen [2014] or Flint and Mare [2016]; In consideration of VRP, $\hat{\alpha} = \ln(\mathbb{E}[\sigma] + VRP) \implies \sigma = e^{\hat{\alpha}} - VRP$.

[10]$price_{Call}^{ATM} \approx price_{Put}^{ATM}$

[11]For the U.S. we have data starting a bit earlier in 2005.

[12]U.S., U.K., France, Germany, Japan, Netherlands, Switzerland. Representing equity indices for which we use implied volatilities are S&P500, FTSE100, CAC40, DAX, Nikkei225, AEX, SMI.

18. Respective regression fit was overall high, with R^2 ranging between 84% and 99% (1^{st} and 3^{rd} quartile). Besides the ex-ante H estimation, we apply ex-post physical long-term memory analysis using simple R/S (Hs; see Hurst [1956]) and empirical Hurst (He; Detrended Fluctuation Analysis, see Weron [2002]), Appendix A.04 further contains tests under corrected R/S (Hrs) and corrected empirical Hurst (Hal) exponents. The reader may address Rea et al. [2009] to find a broad discussion about different ex-post Hurst estimation methodologies.[13] A literature review about persistence analysis on an index level is provided by Caporale et al. [2018], who also explain the procedure of simple R/S analysis in more detail. The reason why we apply an alternative Hurst estimation method (above the most commonly used standard R/S) is to foster robustness of the conclusions we draw on trending behavior of market sentiment.

3.2. Implied Hurst measures Sentiment

First of all, we want to empirically verify our hypothesis that an equity's slope of the implied volatility term structure is a measure of current market mood. To proof this, we compute the implied Hurst exponent H for every country in our data set on a daily basis. From the derived time series, we measure correlations of implied Hurst exponents to several other known sentiment gauges such as volatility indices, consumer confidence and market surveys. Except the volatility indices, all other quotes are less frequent available, e.g. weekly, monthly or quarterly. To avoid over fitting, when estimating correlations always data pairs of the less frequent measure were taken. The example presented in Fig. 1 nicely visualizes first (i) the great concordance of H with already known sentiment measures, and second (ii) the easy and clear interpretability of H, which we see as the great advantage over common volatility indices (such as VIX). In consideration of robustness, each single region's observed correlation of H to VX[14] was highly significant and negative (between -0.43^{***} (Japan) and -0.84^{***} (U.S.)[15]), which clearly supports our sketched hypothesis - if fear (VX) is great, then sentiment (H) is bearish and vice versa. In addition, correlations of H to surveys are also highly significant and support H to be a eligible sentiment measure. In total, for all countries together we compute 71 correlation pairs (H with VX, consumer confidence, investor surveys, fund flows, etc.), from which 63 were of great significance (p-value $< 1\%$), 3 of weak significance (p-value $< 5\%$) and 5 not significant. This observation largely contributes to hypothesis of Fig. 3. An overview of correlation pairs for the different regions is given by the short Table 1, the table documenting all observations together with a variable description is too large to display here and is thus attached at Appendix A.03.

[13] There exist several other studies applying (ex-post) fractal analysis upon return series, examples are Lo [1991]; Peters [1994]; Granger and Hyung [2004] or Alvarez-Ramirez et al. [2008]. Caporale et al. [2018] are - to our knowledge - the only ones, who make similar application of long-term analysis upon implied volatilities.

[14] We use the abbreviation VX for VIX-like implied volatility indices covering regions other than the U.S.

[15] From Pearson's correlation tests.

Table 1
Short-table of correlation pairs between H and regional volatility indices (first row) and to survey based sentiment measures (second row). Every sentiment measure displayed is directional, meaning that they distinguish between bullish and bearish investor mood. All correlation pairs are of high significance (p-value $< 1\%$), confirming that the implied Hurst exponent H is a potent candidate for market mood approximation (Hypothesis 2). Correlations to voltility indices are estimated on a daily basis, to survey based sentiment on a weekly or monthly basis (dependent on data availability). The long-table covering more pairs can be found in Appendix A.03.

correlation of H to ...	US	EUR	FRA	GER	JP	NED	CH	UK
Volatility Index	-0.84	-0.81	-0.84	-0.83	-0.43	-0.77	-0.77	-0.82
Survey based Sentiment[16]	0.87	0.53	0.51	0.47	0.30	0.30	0.47	0.40

3.3. Sentiment Behavior

Now that we found empirical support that H indicates market sentiment, we are interested on how it behaves in its time series. Here, robustness checks were made by direct comparison to VX data. Table 2 highlights that over the entire observed period, sentiment was throughout positive with mean H above 0.5. We further see that the U.S. market was on average more bullish than European countries, which looks like a plausible pattern when considering that European investors faced additional to the financial crises of 2008 the Euro crises around 2011. To get a better understanding of sentiment behavior, we compute log changes in H as $\Delta H_t = ln(\frac{H_t}{H_{t-1}})$, from which's distribution we want to derive insights into over- and under-reactions. All regions realize centered ΔH distributions with means very close to zero, standard deviations are interpreted as nervousness - the less the smaller were the sentiment reactions. Also here, European countries show greater variance than the U.S. The distributions' higher moments further give some interesting insights. All distributions realize heavy tails, this pattern already indicates the anti-persistence of sentiment we are going to examine in more detail later on. This result is interpreted as investors generally tend to overreact. With an eye on skewness,[17] significantly negative for all observed markets, we find that downward moves are of larger sizes than upward moves, meaning that investor confidence is easier destroyed than gained back. The observed pattern is robust across countries, under VX as alternative sentiment gauge and in line with our persistence analysis below. Results of the robustness test using VX are attached in Appendix A.02. The picture of overreaction is in line with the broad consensus from behavioral finance literature (e.g. De Bondt and Thaler [1985, 1987]; Dreman and Lufkin [2000] or Daniel et al. [2002]), but comes from a new perspective.

[16] Data sources for sentiment measures: Investors Intelligence (U.S.), Sentix (Euro Zone; EUR), Banque de France (France; FRA), Fathom (Germany; GER), Sentix (Japan; JP), CBS - Statistics Netherlands (Netherlands; NED), Sentix (Switzerland; CH), Fathom (U.K.)

[17] Skewness significance is tested applying D'Agostino [1970], kurtosis upon Anscombe and Glynn [1983].

Table 2
Statistical moments of the option implied Hurst exponent H for different countries. Mean is throughout above 0.5, indicating that optimistic times overweight pessimistic ones. When comparing European countries with the U.S., we observe that for such the average level is lower, which may be attributable to the fact that these countries additionally faced the Euro-crisis in 2011. Skewness in ΔH is significantly negative, meaning pessimism occurs faster than optimism is raised. Excess kurtosis indicates an overall tendency for overreacting behavior.

	US	EUR	CH	FRA	GER	JP	NED	UK
				H				
Mean	0.59	0.52	0.54	0.52	0.54	0.53	0.53	0.56
Std.Dev.	0.08	0.07	0.07	0.08	0.07	0.07	0.07	0.08
				ΔiH				
Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Std.Dev	0.03	0.06	0.06	0.06	0.04	0.07	0.07	0.05
Skewness	-1.07	-0.17	-0.20	-0.49	-0.29	-0.82	-1.12	-0.55
	***	***	***	***	***	***	***	***
Kurtosis	17.85	9.72	7.21	28.25	7.57	25.38	79.58	12.82
	***	***	***	***	***	***	***	***

* p < 5%, ** p < 1%, *** p < 0.1%

3.3.1. Split into Periods

Contributing to the work of Caporale et al. [2018], we analyze the persistence of investor sentiment over time. In a first step, we concentrate our dataset into two sub-samples representing the periods of (i) financial crises and (ii) post-crises.[18] Similarly to Caporale et al. [2018] we measure persistence of market mood by ex-post realized long-term memory - i.e. realized Hurst exponents of the sentiment time series.[19] Also here, our main sentiment measure is H and robsutness is checked upon VX. To give our study a sounding picture, we use two methods for long-term memory analysis: simple R/S (Hs; cp. Hurst [1956]) and validation upon empirical Hurst exponent (He; Detrended Fluctuation Analysis, cp. Weron [2002]).[20] For the physical Hurst exponents same characteristics hold as for the ex-ante one: ranging between zero and one, if greater 0.5 then the series is trending, if below 0.5 the series is anti-persistent and 0.5 equals classical Brownian motion of no autocorrelation. With a quick look on Figure 4, same pattern can be seen that we thoroughly find from other tests. In line with ΔH's skewness from Table 2, each country's sentiment is more persistent during the period of the financial crises than in normal times - if fear is very present, then investors confirm on the pessimism and do not expect this circumstance to change quickly. In difference, once the confidence was gained back in the latter sub-sample, this optimism showed to be more fragile and anti-persistent, thus investors were more nervous with a tendency to overreac-

[18]From January 2008 to May 2009 for the first period and June 2009 to April 2019 for the latter. Cut-off dates are set in respect to investor sentiment from Figure 1.

[19]Note that ex-post persistence $\neq$ ex-ante persistence; *"persistence of implied persistence"*-analysis

[20]Further results of two additional estimation procedures are attached in Appendix A.04

tion, which may be caused due to the aftermath of the financial crises. Notably, Hurst exponents of the combined sample are below 0.5, thus sentiment over the total horizon is anti-persistent, which goes hand in hand with pattern of excess kurtosis (Table 2) and behavior of market sentiment to overreact. Robustness comes from countrys' VX, where same patterns are observable.

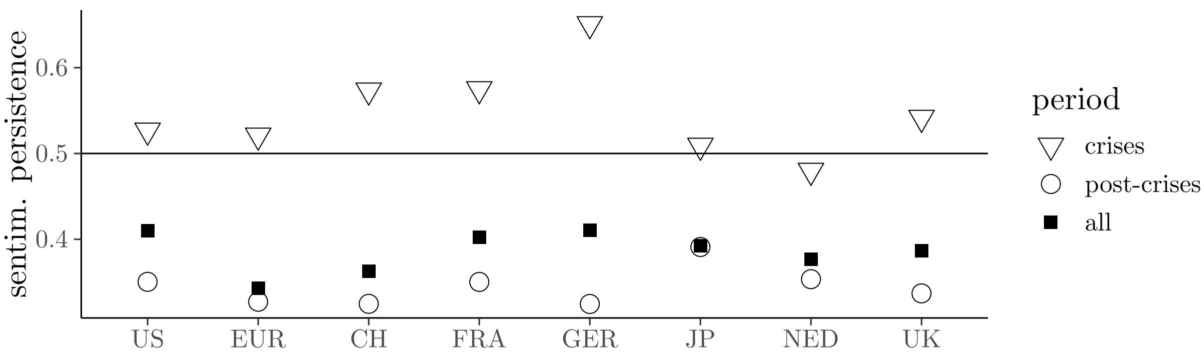

Figure 4
Physical sentiment persistence measured by realized long-term memory (He) upon implied sentiment (ΔH) during periods of financial crises (January 2008 to May 2009), post-crises (June 2009 to April 2019) and the combined sample. Values above 0.5 indicate trending series, below 0.5 reversing behavior. Sentiment is overall anti-persistent, but shows a tendency for trending behavior during crises, such that fear becomes a more persistent market mood than optimism. Robustness is confirmed by using other methodologies for estimating Hurst exponents. Also when directly using implied volatilities as sentiment measure, the pattern preserves, see Table 3 and Appendix A.04.

Table 3
Sentiment persistence measured upon simple R/S analysis (Hs) for the two different periods. Also under Hs, sentiment persistence is typically greater during the crises than afterwards. Confirmation comes from tests upon VX indices. The entire table with all four long-term estimation methods used (Hs, He, Hrs, Hal) can be found in Appendix A.04. No matter which long-term estimator to use, which sentiment measure to examine or at which country to look at, we find that market mood in the period of the financial crises is more persistent than during the time of greater optimism.

	US	EUR	CH	FRA	GER	JP	NED	UK
				ΔH				
crises	0.52	0.40	0.50	0.49	0.52	0.45	0.43	0.52
post- crises	0.41	0.35	0.37	0.42	0.37	0.42	0.37	0.40
tot. sample	0.42	0.37	0.39	0.44	0.44	0.43	0.42	0.40
				ΔVX				
crises	0.51	0.52	0.57	0.48	0.53	0.50	0.55	0.51
post- crises	0.39	0.38	0.40	0.47	0.41	0.41	0.40	0.39
tot. sample	0.40	0.42	0.44	0.48	0.43	0.42	0.42	0.41

3.3.2. Rolling Analysis

Beyond the split of the dataset into two fixed periods, we directly measure the link between sentiment and its persistence on a rolling basis. We choose a time window of one year, for this rolling window we compute the average sentiment $\overline{H}$ and the related persistence of ΔH by simplified R/S analysis. Figure 5 displays the U.S. case of the applied concept, here we additionally show the very similar persistence of ΔVX to foster the applicability of H. From this picture, a tendency can be seen that if the mood gets worse - e.g. around 2009 or 2016 - then persistence of this mood increases. Oppositely, in times of increasing confidence, for example between 2013 to 2015 or 2017 to mid 2018, we find that rising optimism simultaneously occurs with anti-persistence. Therefore, also this analysis confirms our statement that fear is trending and optimism is fragile.

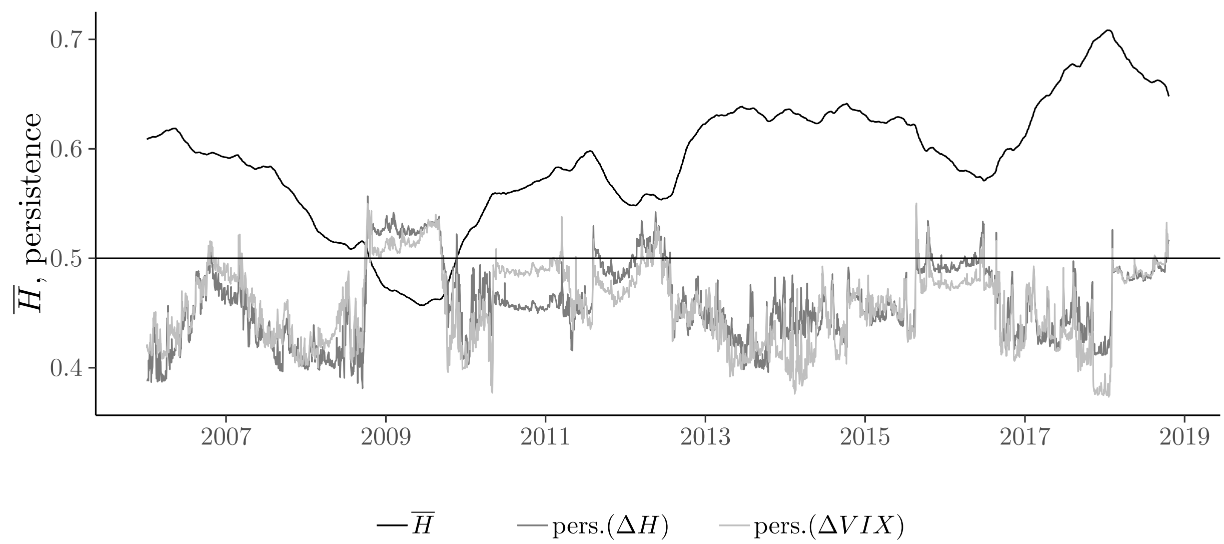

Figure 5
Persistence of U.S. investor sentiment: rolling average sentiment ($\overline{iH}$) versus its corresponding persistence in ΔH in that window, quantified by simplified R/S. Values are right aligned for a rolling window of one year. A tendency can be seen that times with decreasing sentiment are characterized by increasing persistence and vice versa. Thus, when market mood rises, it becomes more fragile ($pers.(\cdot) < 0.5$). When the sentiment moves towards fear, however, it shows that it wants to stay ($pers.(\cdot) > 0.5$). Rolling persistence analysis of ΔVIX confirms the picture.

Following the idea displayed in Figure 5, rolling analysis is made for all eight markets in our dataset. Again, validation of the findings comes from substituting H by VX. To quantify the relation between market mood and its persistence, we establish Pearson's correlation tests. For this purpose we use monthly data pairs to avoid over-fitting, Table 4 gives insights into our analysis output. With exception to the EuroStoxx50 index, correlations are significantly negative under both H and our robustness check of VX, supporting our periodical analysis before.

Table 4
Correlation between 1-year moving average sentiment ($\overline{H}/\overline{VX}$) and its persistence of the same rolling window (measured by R/S analysis; Hs). To avoid over-fitting, always last data-pair per month is used. Significant negative correlation means that during crises fear is trending, but the greater the confidence the less stable it is. Results are broadly in line with our other findings.

cor(sentiment, sentiment persistence)	US	EUR	CH	FRA	GER	JP	NED	UK
$\overline{H}$	-0.31	0.25	-0.50	-0.54	-0.53	-0.40	-0.20	-0.43
	(-4.14)	(3.15)	(-7.00)	(-7.02)	(-7.53)	(-4.87)	(-2.42)	(-5.26)
	***	**	***	***	***	***	*	***
$\overline{VX}$	-0.52	-0.46	-0.50	-0.17	-0.32	-0.18	-0.47	-0.40
	(-6.8)	(-5.75)	(-6.32)	(-1.88)	(-3.71)	(-2.07)	(-5.98)	(-4.77)
	***	***	***	.	***	*	***	***
$\overline{H}$ (under He)	-0.39	-0.09	-0.50	-0.29	-0.31	-0.34	-0.36	-0.27
	(-5.37)	(-1.08)	(-6.91)	(-3.34)	(-4.01)	(-3.97)	(-4.68)	(-3.09)
	***		***	**	***	***	***	**

In a nutshell, from empirical global evidence we first observe that option implied H largely correlates with a broad range of established investor sentiment measures, which confirms our theoretical model. Further, from distributions of this market mood proxy we find patterns of overreactions, especially for the downside. And third, investor sentiment seems to be more stable the greater the pessimism, such that fear is trending and confidence is fragile. Due to the direct (and frequent) measurement of market mood together with its conspicuous behavior, we suggest that these findings give some interesting insights for further research. Because of data availability, H has a another great advantage that similar sentiment analysis can be made on a single stock level, where other sentiment data (like surveys) are scarce.

4. Conclusion

The CBOE VIX - as option implied volatility measure - is broadly used by practitioners to gauge current market fear, but cannot be directly interpreted whether sentiment is bullish or bearish. On the other side, investor surveys evaluate sentiment directly, but are far less frequent, require extensive resources and give solely an incomplete and biased picture of the market. Building on fractional Black-Scholes (Hu and Øksendal [2003]), we hypothesize that the option implied Hurst exponent H can resolve the main drawbacks of both approaches. With an empirical analysis of H for eight different regions around the world, we find interesting features of market sentiment behavior. At all investigated markets, sentiment is significantly negatively skewed, such that fear occurs way faster than confidence is gained back. Further, we parse sentiment's persistence over time in relation to the respective market mood, making use of long-term memory analysis. From these tests, we derive throughout patterns that sentiment persistence negatively correlates with the level of sentiment, meaning that market fear shows tendencies to trend while optimism is typically an anti-persistent occurrence. Robustness is confirmed from various test settings, different markets and validation upon volatility indices.

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Appendix

Appendix A.01: Fractal Brownian Motion

This section provides a short summary of fractional Black-Scholes option pricing as presented in Li and Chen [2014]. Difference between classical Brownian motion and fractal Brownian motion (fBM, denoted as B^H) comes from the covariance, where fBM may allows for serial correlation. B^H is defined as

$$B^H(0,\omega) = 0$$
$$B^H(t,\omega) = \frac{1}{\Gamma(H+\frac{1}{2})}\left[\int_{-\infty}^{0}\left((t-s)^{H-\frac{1}{2}} - (s-)^{H-\frac{1}{2}}\right) dB(s,\omega) + \int_{0}^{t}(t-s)^{H-\frac{1}{2}} dB(s,\omega)\right],$$

with $B(.)$ as classical Brownian motion, $\Gamma(.)$ the gamma function and the Hurst exponent H. H ranges from 0 to 1, $H < 0.5$ implies negative serial correlation, $H > 0.5$ positive one and $H = 0.5$ no autocorrelation, hence under $H = 0.5$ fBM equals classical Brownian motion. Following, expectations of fBM are given by

$$E[B_t^H] = 0 \qquad \forall t \in R$$
$$E[B_t^H B_s^H] = \frac{1}{2}[|t|^{2H} + |s|^{2H} - |t - s^{2H}|]. \qquad \forall t \in R$$
$$E[B_t^H]^2 = t^{2H} \qquad \forall t \in R^+$$

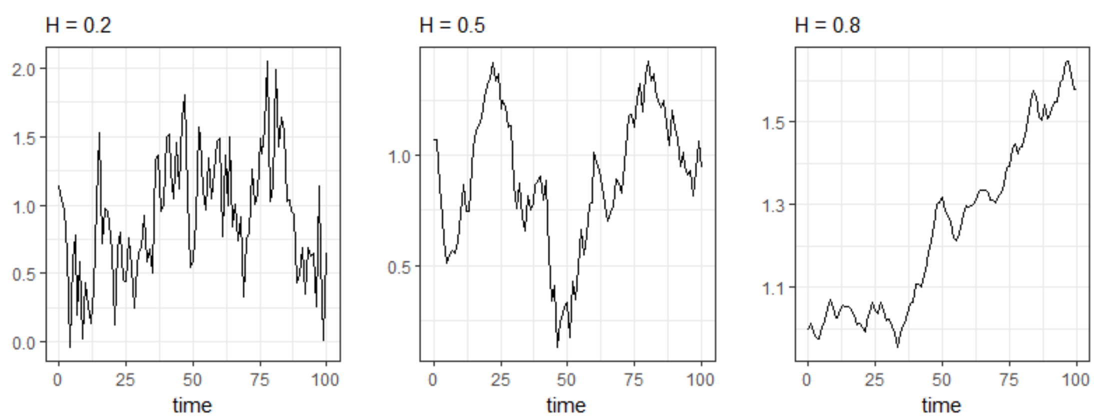

Figure 6
Simulated paths of fractal Brownian motion. Series are once anti-persistent ($H < 0.5$; left), classic Brownian ($H = 0.5$; mid) and trending ($H > 0.5$; right).

The price process under fractional geometric Brownian motion is then

$$dS_t = \mu S_t dt + \sigma S_t dB_t^H,$$

using S, μ, σ as the stock price, fractal drift and fractal volatility. Li and Chen [2014] then show how applying (fractal) Ito's Lemma yields

$$
\begin{aligned}
d\, ln(S_t) &= \mu dt + \sigma dB_t^H - \frac{1}{2}\sigma^2 dt^{2H} \\
ln\left(\frac{S_T}{S_0}\right) &= \mu T - \frac{1}{2}\sigma^2 T^{2H} + \sigma B_T^H,
\end{aligned}
$$

so that the variance can be expressed through

$$
\begin{aligned}
Var\left(ln\left(\frac{S_T}{S_0}\right)\right) &= E\left[ln\left(\frac{S_T}{S_0}\right)^2\right] - E\left[ln\left(\frac{S_T}{S_0}\right)\right]^2 \\
&= \left(\mu T - \frac{1}{2}\sigma^2 T^{2H}\right)^2 + 2\sigma\left(\mu T - \frac{1}{2}\sigma^2 T^{2H}\right) E\left[B_T^H\right] + \sigma^2 E\left[B_T^H\right]^2 \\
&\quad - \left[\mu T - \frac{1}{2}\sigma^2 T^{2H} + \sigma E\left[B_T^H\right]\right]^2
\end{aligned}
$$

which gives us

$$
Var\left(ln\left(\frac{S_T}{S_0}\right)\right) = \sigma^2 T^{2H}.
$$

This relation can then be implemented into classic Black-Scholes option pricing

$$
C_f(S_t, K, \tau, r, \sigma, H) = S_t\, \phi(\hat{d}_1) - K^{-r\tau}\, \phi(\hat{d}_2),
$$

where the Hurst exponent H comes into play at $\hat{d}_1$ and $\hat{d}_2$:

$$
\begin{aligned}
\hat{d}_1 &= \frac{ln\left(\frac{S_t}{K}\right) + r\tau + \frac{1}{2}\sigma_f^2 \tau^{2H}}{\sigma_f \tau^H} \\
\hat{d}_2 &= \hat{d}_1 - \sigma_f \tau^H.
\end{aligned}
$$

Then, the decomposition of observed Black-Scholes implied volatility σ_{BS} into fractal volatility σ_f and H can be denoted as

$$
\sigma_{BS} = \sigma\, \tau^{H-0.5}
$$

so that taking logs, OLS regression can be fit to estimate H from empirical data:

$$
ln(\sigma_{BS}) = ln(\sigma) + (H - 0.5) \cdot ln(\tau) + \epsilon
$$

which we used in Equation 18.

Appendix A.02: Summary Statistics of volatility indices (VIX and other)

Table 5
Summary statistics of market fear measured by country's volatility index. Patterns look similar to H, skewness of ΔVX are significant positive such that jumps for the fear side a larger than for confidence.

	US	EUR	CH	FRA	GER	JP	NED	UK
				VX				
Mean	0.20	0.24	0.19	0.22	0.24	0.24	0.22	0.19
Std.Dev.	0.08	0.09	0.08	0.09	0.09	0.09	0.10	0.08
				ΔVX				
Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Std.Dev	0.07	0.06	0.05	0.06	0.05	0.06	0.06	0.07
Skewness	0.93	0.74	0.59	0.49	0.54	1.39	0.50	0.30
	***	***	***	***	***	***	***	***
Kurtosis	10.17	8.24	6.52	8.01	6.57	12.27	6.61	6.99
	***	***	***	***	***	***	***	***

Appendix A.03: Correlations of implied Hurst to VX and other Sentiment Measures

Table 6
Correlations of H with other sentiment measures for different countries. Aim of this table is to show that the option implied momentum significantly correlates with various other common investor confidence measures, which gives empirical robustness for H to be a valid sentiment measure. For better readability we use simplified alias for country sentiment variables, '.VX' denotes volatility indices for respective countries, '.(*number*)' represent other sentiment measures, the detailed variable description can be found in Table 7.

cor(iH, sentiment)

United States (US)

.VIX	.1	.2	.3	.4	.5	.6	.7	.8	.9
-0.84	0.78	0.71	0.59	0.57	0.51	0.44	0.35	0.22	0.15
***	***	***	***	***	***	***	***	***	*

Euro Zone (EUR)

.VX	.1	.2	.3	.4	.5	.6	.7	.8	.9	.10
-0.81	0.53	0.52	0.46	0.40	0.39	0.37	0.37	0.36	0.35	0.26
***	***	***	***	***	***	***	***	***	***	***

France (FRA)

.VX	.1	.2	.3	.4	.5	.6	.7
-0.84	0.51	0.35	0.34	0.33	0.21	0.17	0.01
***	***	***	***	***	**	*	

Germany (GER)

.VX	.1	.2	.3	.4	.5	.6	.7	.8	.9
-0.83	0.47	0.40	0.38	0.36	0.34	0.31	0.30	0.15	-0.04
***	***	***	***	***	***	***	***	***	

Japan (JP)

.VX	.1	.2	.3	.4	.5
-0.43	0.30	0.19	0.10	-0.29	0.00
***	***	***	***	*	0

Netherlands (NED)

.VX	.1	.2	.3	.4	.5	.6
-0.77	0.30	0.27	0.20	0.18	0.18	0.17
***	***	***	**	**	**	**

Switzerland (CH)

.VX	.1	.2	.3	.4	.5	.6	.7	.8	.9
-0.77	0.47	0.46	0.42	0.41	0.36	0.28	0.25	0.22	0.12
***	***	***	***	***	***	***	***	**	

United Kingdom (UK)

.VX	.1	.2	.3	.4	.5	.6	.7	.8
-0.82	0.40	0.39	0.31	0.24	0.22	0.20	0.20	0.02
***	***	***	***	***	**	**	**	

Table 7

Ticker lexicon and variable description for data used.

Panel A: Countries and used indices

Country	Abbreviation	Index	Country	Abbreviation	Index
United States	US	S&P 500	Japan	JP	NIKKEI 225
Euro Zone	EUR	EuroStoxx 50	Netherlands	NED	AEX
France	FRA	CAC 40	Switzerland	CH	SMI
Germany	GER	DAX	United Kingdom	UK	FTSE100

Panel B: Sentiment measures (Datastream)

Alias	Ticker	Description	Alias	Ticker	Description
US.VIX	CBOEVIX	CBOE Volatility Index	JP.VX	VXJINDX	Volatility Index
US.1	USIISENT	Investor's Intelligence	JP.1	NIKKES1	Sentix Value 1 Month-Nikkei Index
US.2	USSXESIVR	Investors Sentiment (Sentix)	JP.2	NIKKEU1	Sentix Neutral 1 Month-Nikkei Index
US.3	USUMCONSH	Consumer Sentiment Index	JP.3	JPMMFCS	Macromil-Future Cons. Sentim.
US.4	USCCIPSOR	Ipsos Consumer Sentiment Index	JP.4	JPBOJBCFR	Business Sentiment
US.5	USCNFCONQ	Consumer Confidence	JP.5	JPSENTIXR	Investors Sentiment (Sentix)
US.6	USAAII	Sentiment Survey (AAII)	NED.VX	AEXVOLI	Volatility Index
US.7	USTMECO.R	Economic Optimism Index	NED.1	NLCNFBUSQ	Business Confidence, surveys
US.8	USFCFB.IA	Mutual Fund Flows - Bonds	NED.2	NLEUSESIG	EC Economic Sentiment Indicator
US.9	USFCFE.QA	Mutual Fund Flows - Equity	NED.3	NLCNFCONR	CBS Consumer Confidence
EUR.VX	VSTOXXI	Volatility Index	NED.4	NLOCS002Q	EC Consumer Confidence
EUR.1	EMSXESFPR	Economic Sentiment (Sentix)	NED.5	NLML2038Q	CLI Consumer Confidence
EUR.2	EMSXESF.R	Euro-Zone Econ. Sentiment (Sentix)	NED.6	NLCNFCONQ	CBS Consumer Confidence (SA)
EUR.3	EMSXESISR	Investor Sentiment (Sentix)	CH.VX	VSMIIDX	Volatility Index
EUR.4	EMZEWES.R	Economic Sentiment (ZEW)	CH.1	SWSXESF.R	Economic Sentiment (Sentix)
EUR.5	EMEUSESIG	EC Economic Sentiment Indicator	CH.2	SWSXESFIR	Econ. Sent., Institutional (Sentix)
EUR.6	EKCNFBUSQ	EC Industrial Confidence Indicator	CH.3	SWSXESFPR	Econ. Sent., Private (Sentix)
EUR.7	EKEUSESIG	EC Euro-Zone Econ. Sent.	CH.4	SWSXESISR	Investors Sentiment (Sentix)
EUR.8	EMECOIN.Q	Euro Area Business Cycle	CH.5	SWQL2038Q	CLI Consumer Confidence
EUR.9	EKCNFCONQ	Consumer Confidence, survey	CH.6	SWSXESNIR	Current Sit. Survey, Inst. (Sentix)
EUR.10	EK45.99BQ	EC Construction Confidence	CH.7	SWSXESN.R	Current Sit. Survey (Sentix)
FRA.VX	CACVOLI	Volatility Index	CH.8	SWSXESNPR	Current Sit. Survey, Priv. (Sentix)
FRA.1	FRSURCBSQ	Business Climate Indicator	CH.9	SWQCS002Q	EC Consumer Confidence
FRA.2	FRFTMESIR	Fathom Econ. Sentiment	UK.VX	VFTSEIX	Volatility Index
FRA.3	FRINDSYNQ	Composite Business Climate	UK.1	UKEUSESIG	EC Economic Sentiment Indicator
FRA.4	FREUSESIG	EC Economic Sentiment	UK.2	UKFTMESIR	Fathom Economic Sentiment
FRA.5	FRSURBSSQ	Business Sentiment Indicator	UK.3	UKLBBOVUR	Lloyds Business Barometer
FRA.6	FRCCIPSOR	Ipsos Primary Consumer Sentiment	UK.4	UKOL2038Q	CLI Consumer Confidence
FRA.7	FRSEBIIHR	France, Euro Break-Up Index	UK.5	UKGFKCCNR	GFK Consumer Confidence
GER.VX	VDAXNEW	Volatility Index	UK.6	UKCNFCONQ	EC Consumer Survey
GER.1	BDFTMESIR	Fathom Econ. Sentiment Indicator	UK.7	UKOCS002Q	EC Consumer Confidence
GER.2	BDSXESISR	Sentix Investors Sentiment	UK.8	UKCCIPSOR	Ipsos Primary Consumer Sentiment
GER.3	BDSXESF.R	Sentix Economic Sentiment			
GER.4	BDEUSESIG	EC Economic Sentiment			
GER.5	BDSXESFIR	Sentix Econ. Sent. (Institutional)			
GER.6	BDZEWECSR	ZEW Economic Expectations			
GER.7	BDSXESN.R	Sentix Current Situation Surveys			
GER.8	DAXINU1	Sentix 1 M-DAX Index			
GER.9	BDCCIPSOR	Ipsos Primary Consumer Sentiment			

Appendix A.04: Sentiment persistence during periods

Table 8
Persistence analysis of investor sentiment as measured by long term-dependence for different countries. Hs (simplified R/S), Hrs (corrected R/S), He (empirical Hurst) and Hal (corrected empirical H.) are Hurst exponents under different methodologies, estimation details for can be found in Weron [2002]. This table gives robustness for two claims. First, that H represents investor sentiment, which is here confirmed by the very similar patterns between H and the broadly accepted volatility indices (e.g. VIX). Second, sentiment persistence is greater during crises than during more confident times.

Panel A: Persistence of ΔiH.

	US	EUR	CH	FRA	GER	JP	NED	UK
crises (2008/2009-05)								
Hs	0.52	0.40	0.50	0.49	0.52	0.45	0.43	0.52
Hrs	0.57	0.42	0.56	0.52	0.58	0.45	0.46	0.55
He	0.53	0.52	0.57	0.58	0.65	0.51	0.48	0.54
Hal	0.47	0.44	0.51	0.52	0.59	0.44	0.42	0.48
post crises (2009-06/)								
Hs	0.41	0.35	0.37	0.42	0.37	0.42	0.37	0.40
Hrs	0.39	0.34	0.36	0.39	0.35	0.41	0.36	0.38
He	0.35	0.33	0.32	0.35	0.32	0.39	0.35	0.34
Hal	0.32	0.29	0.29	0.32	0.29	0.36	0.32	0.31
all (2008/2019-04)								
Hs	0.42	0.37	0.39	0.44	0.44	0.43	0.42	0.40
Hrs	0.41	0.36	0.37	0.42	0.43	0.42	0.41	0.39
He	0.41	0.34	0.36	0.40	0.41	0.39	0.38	0.39
Hal	0.38	0.31	0.33	0.37	0.38	0.36	0.34	0.36

Panel B: Persistence of ΔVX.

	US	EUR	CH	FRA	GER	JP	NED	UK
crises (2008/2009-05)								
Hs	0.51	0.52	0.57	0.48	0.53	0.50	0.55	0.51
Hrs	0.56	0.57	0.64	0.52	0.60	0.57	0.62	0.56
He	0.64	0.70	0.71	0.64	0.72	0.68	0.70	0.54
Hal	0.57	0.64	0.66	0.58	0.66	0.61	0.64	0.49
post-crises (2009-06/)								
Hs	0.39	0.38	0.40	0.47	0.41	0.41	0.40	0.39
Hrs	0.37	0.35	0.38	0.46	0.39	0.40	0.38	0.36
He	0.32	0.32	0.37	0.41	0.36	0.40	0.34	0.33
Hal	0.30	0.30	0.34	0.38	0.34	0.37	0.31	0.30
all (2008/2019-04)								
Hs	0.40	0.42	0.44	0.48	0.43	0.42	0.42	0.41
Hrs	0.38	0.39	0.42	0.46	0.41	0.42	0.40	0.38
He	0.37	0.36	0.41	0.42	0.41	0.40	0.38	0.36
Hal	0.34	0.34	0.38	0.38	0.38	0.37	0.35	0.33

Appendix A.05: Sentiment correlation across countries

Table 9
Correlations of H. Market sentiment is throughout positive correlated across the globe. Non-surprisingly, sentiment of European countries tends to evolve more similar among each other than to countries of other continents. From this sample, Japan seems to be the most unaffected market when looking on average sentiment correlations to other countries (ex.EUR, so that average is built only from countries). On a continental perspective, similar patterns are observable, European countries tend to be higher correlated with the U.S. and less correlations are derived for other continents, which we thus see as slightly more isolated/independent.

	US	EUR	CH	FRA	GER	JP	NED	UK
US	1	0.75	0.77	0.82	0.76	0.35	0.77	0.88
EUR		1	0.79	0.88	0.90	0.42	0.87	0.82
CH			1	0.84	0.81	0.50	0.82	0.84
FRA				1	0.89	0.40	0.86	0.87
GER					1	0.42	0.89	0.82
JP						1	0.42	0.45
NED							1	0.83
UK								1
∅ (ex. EUR)	0.72		0.76	0.79	0.76	0.42	0.77	0.78

Appendix A.06: Correlation Plot, Sentiment vs. Sentiment Persistence

Figure 7
Correlation plot of U.S. market sentiment as by $\overline{iH}$ (rolling 1 year average to represent period) to sentiment persistence for same rolling window. Persistence is measured by long-term analysis computing simple R/S Hurst exponent of ΔiH (left plot) and to support robustness for ΔVIX. Solely end-of-month observations are used to avoid over-fitting.